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DIFFRACTION OF A SHORT WAVE AT THE JUNCTION BETWEEN TWO PLATES, ONE OF WHICH IS REINFORCED BY RIBS†

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The two-dimensional problem of the diffraction of a plane hydroacoustic wave at the junction between two elastic plates is considered. One of the plates is uniform and the other is reinforced with a periodic set of uniform stiffeners. The plates are joined in such a way that together they form a plane which splits the space into two subspaces. An acoustic medium fills one of these half-spaces. It is assumed that one cannot ignore reflection from the surface of the stiffeners nor neglect their moment impedance. The conditions at the junction of the plates is not completely fixed, i.e. the general solution of the problem is investigated. A modified stationary-phase method is used to investigate the solution obtained and enables the asymptotic forms of the field in the neighbourhood of particular directions to be obtained. © 1997 Elsevier Science Ltd. All rights reserved.

1. FORMULATION OF THE PROBLEM

We will assume that an acoustic medium fills the upper half-plane $-\infty < x < \infty$. The acoustic-pressure field $p(x, y)$ satisfies the Helmholtz equation

$$(\Delta + k^2)u = 0 \tag{1.1}$$

Here $k^2 = \omega^2/c^2$ is the wave number of the oscillations of the medium, ω is the oscillation frequency, and c is the velocity of sound in the medium. The two semi-infinite elastic plates are situated on the boundary $-\infty < x < \infty, y = 0$. The oscillations of the plate, which occupy the left half-axis, are described by Kirchhoff's equation

$$L_1 p(x) = (\partial_x^4 - k_1^4)p_y(x) + \nu_1 p(x) = 0 \tag{1.2}$$

$$-\infty < x < 0, \quad y = 0$$

$$k_1^4 = M_1\omega^2/D_1, \quad \nu_1 = \rho_0 c^2/D_1$$

where ∂_x is the derivative with respect to x, $p_y(x)$ is the value of the field $p(x, y)$ on the plate, k_1 is the wave number of the plate oscillations, M_1 is the surface density, D_1 is the cylindrical stiffness of the plate, ρ_0 is the density of the acoustic medium, $p_y(x)$ is the derivative with respect to y of $p(x, y)$ on the boundary, and L_1 is a differential operator, defined by (1.2). The plate, situated on the right semi-axis, is reinforced with a periodic set of stiffeners at the points $x = na, y = 0$, where n is any natural number and a is a fixed real number, equal to the distance between neighbouring stiffeners. Kirchhoff's equation

$$Lu(x) = (\partial_x^4 - k_0^4)p_y(x) + \nu p(x) = 0 \tag{1.3}$$

$$0 < x < \infty, \quad y = 0$$

$$k_0^4 = M\omega^2 / D, \quad \nu = \rho_0\omega^2 / D$$

is satisfied everywhere on the right semi-axis apart from these points. Here k_0 is the wave number of the plate oscillations, M is the surface density, D is the cylindrical stiffness of the plate, and L is the differential operator defined by (1.3).

We will assume, in addition, that the function $p_y(x)$ is continuous together with its first derivative $p'_y(x)$ with respect to x everywhere on the x axis. Physically, this condition indicates that the displacements

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and rotations of the plates are continuous at all points. Further, the function $p''_y(x)$ is continuous everywhere on the x axis with the exception of the point $x = 0$, and $p''_y(x)$ is continuous everywhere, apart from the points $x = 0, x = na$, where the following condition is satisfied at the points $x = na$ (see [1])

$$[p'''_y(x)](na) = Zp_y(na) \tag{1.4}$$

where $[f(x)]$ is the jump in the function f on passing through the point $x = na$. The condition for p''_y to be continuous corresponds to the absence of a bending moment acting from the side of the ribs on the plate, while condition (1.4) indicates that there is no jump in the shear forces on passing through a rib.

We will obtain the so-called general solution of the problem below. It contains a certain number of arbitrary constants. These constants can be found after substituting the contact conditions at the junction of the plates. It is assumed that the field $p(x, y)$ satisfies the Meixner conditions on the ribs and at the junction of the plates. The solution of the problem satisfies the limiting absorption principle. All the calculations are carried out assuming that Im $k^2 > 0$, and in the final formulae we take the limit as Im $k^2 \to 0$.

We will seek a solution of problem (1.1)–(1.4) in the form

$$p(x,y) = p_0(x,y) + u(x,y), \quad p_0(x,y) = p_i(x,y) + p_r(x,y) \tag{1.5}$$

Here

$$p_i(x,y) = e^{-ik(xc_0+ys_0)}, \quad c_0 = \cos\varphi_0, \quad s_0 = \sin\varphi_0 \tag{1.6}$$

represents an incident plane wave, φ_0 is the angle of incidence, $p_r(x, y)$ is the field reflected by the uniform plates, and $p_r(x, y)$ satisfies the Helmholtz equation and boundary conditions (1.2) and (1.3) for all values of x. By [2]

$$p_r(x,y) = \mathrm{R}e^{-ik(xc_0-ys_0)} + \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\lambda}{w(\lambda)}e^{i(\lambda x+\gamma(\lambda)y)}\left(C_0 + i\lambda C_1 + (R_1 - R)\frac{iw(-kc_0)}{\lambda + kc_0}\right) \tag{1.7}$$

$$R = \frac{S_-(k_0,\nu)}{S_+(k_0,\nu)}, \quad R_1 = \frac{S_-(k_1,\nu_1)}{S_+(k_1,\nu_1)}$$

$$S_\pm(\xi,\eta) = ikxc_0(k^4c_0^4 - \xi^4) \pm \eta$$

The first term represents the field reflected by the uniform right plate, infinite on both sides, and R is the reflection coefficient [1]. The second term takes into account the effect of the left plate and the junction, and R_1 is the reflection coefficient of the left uniform plate. The first two terms in the integral describe the field scattered by the joint: C_0 and C_1 are undetermined constants, about which we spoke above, and which can be found from the conditions at the junction of the plates. Further

$$w(\lambda) = l_1^+(\lambda)l^-(\lambda)$$

where $l^\pm$ and $l_1^\pm$ are the result of factorizing the symbols of the boundary operators $l(\lambda)$ and $l_1(\lambda)$ of the right and left plates, respectively (see (1.2) and (1.3)).

In (1.5) $u(x, y)$ satisfies the Helmholtz equation, boundary conditions (1.2) and (1.3) and the conditions on the ribs

$$[\partial_x^3 u_y](na) = Zp_y(na) \tag{1.8}$$

where the square brackets denote jumps in the function on passing through the point $x = na$ (see (1.4)).

In addition, the field $u(x, y)$ satisfies the limiting absorption principle and the Meixner conditions on the ribs.

2. THE SYSTEM OF EQUATIONS FOR THE BOUNDARY-CONTACT CONSTANTS

The solution of (1.1)–(1.3), (1.8) can be obtained from the condition

$$u(x,y)=\sum Zu_y(na)g(x,y;na) \tag{2.1}$$

Here and everywhere henceforth summation is carried out from $n = 1$ to $n = \infty$; $g(x, y; x'), x' > 0$ is the solution of the Helmholtz equation with the boundary conditions

$$L_1 g(x,y;x')=0,\ x<0,\ Lg(x,y;x')=\delta(x-x'),\ x>0$$

We can equate some of the arbitrary constants in the solution of this problem to zero.

Using the Wiener–Hopf method, we obtain

$$g(x,y;x')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\lambda}{w(\lambda)}e^{i(\lambda x+\gamma(\lambda)y)}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dt}{\lambda-t}\frac{w(t)}{l(t)}e^{-itx'} \tag{2.2}$$

The integration contours pass above the singularities of the integrand, which lie on the negative semi-axis, and below the singularities which lie on the positive semi-axis (with the exception of the point $\lambda = t$). The contour in the inner integral passes round the point $\lambda = t$ from above.

The right-hand side of (2.1) contains unknown values of the normal derivative of the acoustic field on the ribs $x = na, y = 0$. It is sufficient to obtain these values in order to obtain a solution of problem (1.1)–(1.4).

Differentiating both sides of (2.1) with respect to y and assuming that $x = ma, y = 0$, we can obtain an infinite system of linear algebraic equations for the unknowns $u_y(na)$. However, it is much more convenient to introduce the new unknown quantities

$$v_m=(p_0+u)_y(ma),\quad m>0$$

The system of equations for v_m has the form

$$v_m-\sum Zv_n g_y(ma,0;na)=p_{0y}(ma),\quad m>0 \tag{2.3}$$

3. INVESTIGATION OF THE SYSTEM

We will consider (2.3) as an equation in space ℓ_2 of sequences of complex number, where we will assume that $\operatorname{Im} k^2 > 0$, i.e. we have not transferred to limiting absorption.

Consider the structure of the matrix $\{g_y(ma, 0; na)\}$ of system (2.3). From (2.2) we have asymptotically for large values of ka

$$\begin{aligned}g_y(ma,0;na)&=i\gamma(\chi)\alpha e^{i\chi|m-n|a}+i\gamma(\chi)\alpha_1 e^{i\chi(m+n)a}+tO((k|m-n|a)^{-3/2})+\\&+q_m O((kna)^{-3/2})+r_n O((kma)^{-3/2})\end{aligned} \tag{3.1}$$

$$\left(\alpha=\frac{i}{l'(\chi)},\quad \alpha_1=\frac{\alpha}{2\chi}\frac{w(-\chi)}{w'(\chi)}\right)$$

with certain limits t, q_m and r_n. Here χ is the positive root of $1(\lambda)$, and the prime on $l(\lambda)$ and $w(\lambda)$ denotes a derivative with respect to λ. The remaining terms in this asymptotic form correspond to the matrix of a certain limited operator in ℓ_2, which is small in norm for large ka.

The second relation of (1.5) and Eqs (1.6) and (1.7) give

$$p_{0y}(ma)=pe^{-ikmac_0}+qe^{i\chi ma}+O((kma)^{-3/2}) \tag{3.2}$$

$$p=ik(R-1)s_0,\quad q=-\frac{\gamma(\chi)}{w'(\chi)}\left(C_0+i\chi C_1+(R_1-R)\frac{iw(-kc_0)}{\chi+kc_0}\right)$$

Despite the fact that the asymptotic forms (3.1) and (3.2) are written for real k, they retain their form for complex k also, such that $\operatorname{Im} k^2 > 0$. All the terms here decrease exponentially, while the residual terms decrease more rapidly than the main terms.

Bearing Eqs (3.1) and (3.2) in mind, we will rewrite system (2.3) in the form of an equation in ℓ_2 space

$$v - Bv - Cv - \varepsilon B^{(1)}v = A + \varepsilon A^{(1)}, \quad \varepsilon = (ka)^{-3/2} \tag{3.3}$$

Here B is an operator whose matrix has the form $\{i\gamma(\chi)\alpha e^{i\chi|m-n|a}\}_{m,n=1}^{\infty}$, the matrix of the operator C is $\{i\gamma(\chi)\alpha_1 e^{i\chi|m+n|a}\}_{m,n=1}^{\infty}$, the operator $\varepsilon B^{(1)}$ is formed by the residual terms in (3.1), A is a sequence of elements of the form $\{pe^{-ikmas_0+q}e^{i\chi ma}\}_{m=1}^{\infty}$, and $\varepsilon A^{(1)}$ is a sequence formed from the residual terms of (3.2).

In addition to (3.3) we will consider an equation of the form

$$\tilde{v} - B\tilde{v} - C\tilde{v} = A \tag{3.4}$$

We will apply the theory of approximate methods [3] to the pair of equations (3.3) and (3.4).

Subtracting (3.4) from (3.3) we obtain that the following equation must be satisfied for the difference $u = v - \tilde{v}$ between the exact and approximate solutions

$$u - Bu - Cu = \varepsilon(B^{(1)}(u + \tilde{v}) + A^{(1)}) \tag{3.5}$$

Solving the approximate equation (3.4) we obtain an explicit expression for the operator R, which is the inverse of the operator $(1 - B - C)$. Equation (3.5). Equation (3.5) can be converted to the form

$$u = \varepsilon R(B^{(1)}(u + \tilde{v}) + A^{(1)}) \tag{3.6}$$

This equation can be solved by iterations if the condition for the Neyman series to converge

$$q = 1 - \varepsilon \| R \| \| B^{(1)} \| > 0 \tag{3.7}$$

is satisfied.

Hence, when condition (3.7) is satisfied, the unique solvability of the exact equation (3.3) follows from the unique solvability of the approximate equation (3.4). Moreover, the estimate of the accuracy of the approximation employed

$$\| u \| \leq \varepsilon \| R \| (\| B^{(1)} (\| u \| + \| \tilde{v} \| + \| A^{(1)} \|)$$

i.e.

$$\| u \| \leq q^{-1} \varepsilon \| R \| (\| B^{(1)} \| \| \tilde{v} \| + \| A^{(1)} \|)$$

follows from (3.6).

Hence, when condition (3.7) is satisfied the accuracy of the approximation is proportional to ε.

We will henceforth retain the notation v for the approximate solution.

4. SOLUTION OF THE SYSTEM

We will separate the principal part of the system in the short-wave limit as $ka \to \infty$. We obtain from (2.3)

$$v_m - \sum Bv_n e^{i\chi|m-n|a} = He^{i\chi ma} + pe^{-ikmac_0}, \quad m > 0 \tag{4.1}$$

$$B = i\gamma(\chi)\alpha Z, \quad H = \sum Zv_n e^{i\chi na} i\gamma(\chi)\alpha_1 + q \tag{4.2}$$

Note that H is an unknown quantity.

The solution of a system of the form (4.1) by the Wiener–Hopf method is known [4]. Hence, for the function

$$V(\zeta)=v_1\zeta+v_2\zeta^2+\ldots$$

in the unit circle $|\zeta|=1$ we obtain the expression

$$V(\zeta)=\frac{\zeta}{\zeta-\theta^{-1}}\left(H\frac{e^{i\chi a}-e^{-i\chi a}}{e^{-i\chi a}-\theta}+p\frac{e^{i\chi a}-e^{ikac_0}}{e^{ikac_0}-\theta}\frac{\zeta-e^{-i\chi a}}{\zeta-e^{ikac_0}}\right) \tag{4.3}$$

Here θ is the Floquet factor, which is the root of the dispersion quadratic equation

$$\theta^2-2(\cos\chi a+iB\sin\chi a)\theta+1=0$$

which is less than unity in absolute value when $\operatorname{Im}k^2>0$.

To obtain the unknown constant H, it is sufficient to put $\zeta=e^{i\chi a}$ in (4.3) and use (4.2) (see [4]). We obtain

$$H=\left[Zi\gamma(\chi)\alpha_1 pe^{i\chi a}\frac{e^{i\chi a}-e^{-i\chi a}}{e^{-ikac_0}-\theta}+q(e^{i\chi a}-\theta^{-1})\right]\times$$

$$\times\left[e^{i\chi a}-\theta^{-1}-Zi\gamma(\chi)\alpha_1 e^{i\chi a}\frac{e^{i\chi a}-e^{-i\chi a}}{e^{-i\chi a}-\theta}\right]^{-1}$$

These calculations become incorrect in the neighbourhood of those frequencies for which the frequency χ of the surface wave is a multiple of the Floquet frequency ($\chi=-\tau+2\pi m/a$) or the frequency of the projection of the incident wave ($\chi=kc_0+2\pi n/a$), where m and n are certain integers, while τ is a real number such that $\theta=e^{i\tau a}$. Similar resonance phenomena are known in the theory of periodically reinforced plates and were pointed out, for example, in [5].

Taking (4.3) into account we obtain from (2.1) an expression for the first term in the short-wave asymptotic form of the diffracted field

$$u(x,y)\sim\frac{Z}{2\pi}\int_{-\infty}^{\infty}\left(\frac{V(e^{-i\lambda a})}{l(\lambda)}+\frac{V(e^{i\chi a})w(-\chi)}{l'(\chi)w(\lambda)(\lambda+\chi)}\right)e^{i(\lambda x+\gamma(\lambda)y)}d\lambda,\quad ka\to\infty \tag{4.4}$$

5. INVESTIGATION OF THE SOLUTION

Expression (4.4) for the total field $u(x,y)$ can be uniquely split into two terms

$$u(x,y)\sim u_s(x,y)+u_0(x,y),\quad ka\to\infty$$

where

$$u_s(x,y)=\frac{Z}{2\pi}\int_{-\infty}^{\infty}\frac{V(e^{-i\lambda a})}{l(\lambda)}e^{i(\lambda x+\gamma(\lambda)y)}d\lambda$$

$$u_0(x,y)=\frac{Z}{2\pi}\frac{w(-\chi)}{l'(\chi)}V(e^{i\chi a})\int_{-\infty}^{\infty}\frac{e^{i(\lambda x+\gamma(\lambda)y)}}{w(\lambda)(\lambda+\chi)}d\lambda$$

(the first term is equal to the sum of the fields diffracted by the ribs, while the second describes the field diffracted by the joint due to the action of the overall surface wave propagating from the ribs).

We can calculate the field in the far zone, due to the presence of the second term, in the usual way. It has the form of a cylindrical wave propagating from the joint.

We will now investigate the structure of the field $u_s(x,y)$ diffracted by the ribs. In accordance with expression (3.4) it can also be split into two terms

$$u_s(x,y)=u_s^{(1)}(x,y)+u_s^{(2)}(x,y) \tag{5.1}$$

$$u_s^{(1)}(x,y)=\frac{h}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\lambda a}}{e^{-i\lambda a}-e^{-i\tau a}}\frac{e^{i(\lambda x+\gamma(\lambda)y)}}{l(\lambda)}d\lambda \tag{5.2}$$

$$u_s^{(2)}(x,y)=\frac{P}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\lambda a}}{e^{-i\lambda a}-e^{-i\tau a}}\frac{e^{-i\lambda a}-e^{-i\chi a}}{e^{-i\lambda a}-e^{-ikac_0}}\frac{e^{i(\lambda x+\gamma(\lambda)y)}}{l(\lambda)}d\lambda$$

$$h=ZH\frac{e^{i\chi a}-e^{-i\chi a}}{e^{-i\chi a}-e^{i\tau a}},\quad P=Zp\frac{e^{i\chi a}-e^{ikac_0}}{e^{ikac_0}-e^{i\tau a}}$$

The term $u_s^{(1)}(x,y)$ corresponds to the field due to the action on the ribs of the surface wave propagating from the joint.

We will use the method of steepest descents to calculate the far field for $u_s^{(1)}(x,y)$. We will first convert the integral by making the change of variable $\lambda = k\cos t$. Changing to polar coordinates r, φ, we obtain from (5.2)

$$u_s^{(1)}(r,\varphi)=-\frac{h}{2\pi}\int_{\Gamma}k\sin t\frac{e^{ikr\cos(\varphi-t)}}{l(k\cos t)}\frac{e^{-ika\cos t}}{e^{-ika\cos t}-e^{-i\tau a}}dt \tag{5.3}$$

The integration contour Γ consists of the sections $(-\pi-i\infty,\pi]$, $[\pi,0]$, $[0,i\infty)$.

The integrand in (5.3) has singularities at the points $t=\varphi_n$, such that

$$k\cos\varphi_n=\tau+2\pi n/a \tag{5.4}$$

for any integer n. For certain values of n, these singularities may lie close to the real axis. The steady contour Γ_{st} for integral (5.3) intersects the real axis t at the saddle point $t=\varphi$, which lies in the section $(0,\pi)$. When the contour Γ is deformed into the steady contour Γ_{st}, the poles of the integrand in (5.3) may be intersected.

We will follow the change in the field pattern $u_s^{(1)}(x,y)$ in the far zone as the angle of observation φ changes. We will first assume that we are far from the joint of the plates and hence the initial angle is greater than the largest of the values of φ_n (but not too close to π). Then, when the contour Γ is deformed into the contour Γ_{st} not one of the poles lying on the real axis is intersected. Hence, the contribution of this zone in the far field gives only a saddle point. Hence, the far field $u_s^{(1)}(x,y)$ is a cylindrical wave propagating from the joint. We will gradually reduce φ. Beginning from the instant when φ falls in the δ-neighbourhood of the angle φ_*, equal to the greatest of the real values of the angles φ_n, which satisfying condition (5.4), the form of the field $u_s^{(1)}(x,y)$ changes.

We will use the saddle-point method, modified taking into account the closeness of the location of the saddle point and the pole [6]. For simplicity we will confine ourselves to a small δ-neighbourhood of the angle φ_*. Using formula (3.34) from [7], in the limit as $\delta\to 0$ we obtain

$$u_s^{(1)}(r,\varphi)\sim\frac{-he^{ikr}}{kal(k\cos\varphi_*)}U_\delta \tag{5.5}$$

$$\varphi=\varphi_*+\delta,\quad kr\to\infty,\quad \delta\to 0$$

$$U_\delta=\frac{k}{2i}e^{-ikr\delta^2/2}\left[1+(1-i)\left(C\left(\sqrt{\frac{kr}{\pi}}\delta\right)-iS\left(\sqrt{\frac{kr}{\pi}}\delta\right)\right)\right]$$

Here $C(x)$ and $S(x)$ are Fresnel integrals.

The structure of expression (5.5) is similar to the well-known asymptotic forms (see for example, (3.36)–(3.41) in [7]) for the diffraction field at a wedge in the neighbourhood of the light-shadow boundary. This indicates that the field $u^{(1)}{}_s$ in the neighbourhood of φ_* and the field in the neighbourhood of the light-shadow boundary in the case of diffraction by a wedge are similar.

When the angle φ is reduced further the contribution from the point φ_* is taken into account by a term equal to the residue of the integrand at the pole $\varphi=\varphi_*$, since when the initial contour Γ is deformed into the contour Γ_{st} this pole will be intersected. For all $\varphi>\varphi_*$ this contribution to $u_s^{(1)}$ will be present in the form of the term

$$-ihk\sin\varphi_*\frac{e^{ikr\cos(\varphi-\varphi_*)}}{l(k\cos\varphi_*)}e^{-ika\cos\varphi_*} \tag{5.6}$$

Hence, when $\varphi<\varphi_*$, the field $u_s^{(1)}$ in the far zone contains a wave of the form (5.6) which does not decrease as r increases, and so on. As the angle φ increases the contour Γ_{st} will sequentially intersect the poles φ_n of the integrand, and all new terms of the form (5.6) corresponding to real values of φ_n will be added to $u^{(1)}_s$. The change in the field $u^{(1)}_s$ when φ passes through the pole φ_n is described by expressions of the form (5.5), containing Fresnel integrals. Hence, the general expression for the field $u_s^{(1)}$ outside of directions in which amplification occurs has the form of the sum of terms of the form (5.6), apart from terms $O((kr)^{-1/2})$, taken over all n such that $0 < \varphi < \varphi_n < \pi$.

We can similarly investigate the term $u^{(2)}_s$ in (5.1), corresponding to the field caused directly by the incidence of a plane wave on a rib. Here there are two infinite series of singularities of the integrand. These are poles are the points $\lambda = \tau + 2\pi n/a$ with integer n, and also poles at the points $\lambda = -kc_0 + 2\pi m/a$ with integer m. We will assume, for simplicity, that the singularities of these two series do not coincide.

In the neighbourhood of the angles $\varphi = \varphi_n + \delta$ the field $u_s^{(2)}$ makes an additional contribution to $u_s^{(1)}$, namely

$$u_s^{(2)}(r,\varphi) \sim \frac{-Pe^{ikr}}{kal(k\cos\varphi_n)} \frac{e^{-ika\cos\varphi_n} - e^{-i\chi a}}{e^{-ika\cos\varphi_n} - e^{ikac_0}} \frac{k}{2i} e^{-ikr\delta^2/2} U_\delta$$
$$\varphi = \varphi_n + \delta, \quad kr \to \infty, \quad \delta \to 0$$

In addition, the field $u_s^{(2)}$ also has a number of directions $\varphi = \vartheta_m + \delta$ in which it increases, where

$$\vartheta_m = -kc_0 + 2\pi m / a \tag{5.7}$$

To fix our ideas we will assume that the angle of incidence φ_0 is greater than $\pi/2$. Proceeding in the same way as in the previous case, we obtain that the far field in the direction of the angle φ is the sum of terms of the form

$$\frac{iP(e^{-ika\cos\vartheta_m} - e^{-i\chi a})}{l(k\cos\vartheta_m)} \frac{e^{-ika\cos\vartheta_m}}{e^{-ika\cos\vartheta_m} - e^{-i\tau a}} k\sin\vartheta_m e^{ikr\cos(\varphi-\vartheta_m)}$$

with ϑ_m, which satisfies condition (5.7), such that $0 < \varphi < \vartheta_m < \pi$. If φ lies in the δ-neighbourhood of the angle ϑ_m, we have

$$u_s^{(2)}(r,\varphi) \sim \frac{-Pe^{-ikr}}{kal(k\cos\vartheta_m)} \frac{e^{-ika\cos\vartheta_m} - e^{-i\chi a}}{e^{-ika\cos\vartheta_m} - e^{-i\tau a}} \frac{k}{2i} e^{-ikr\delta^2/2} U_\delta$$
$$\varphi = \vartheta_m + \delta, \quad kr \to \infty, \quad \delta \to 0$$

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